Fast Computation of MoM Matrix Elements over a Wide Frequency Range using a New Interpolation Technique

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Abstract — **This paper presents an interpolation algorithm, in the context of Method of Moments (MoM), for computing the matrix elements for planar microstrip structures over a wide frequency range. Three different interpolating functions are implemented depending upon the distance between the basis and the testing functions. It is demonstrated that the use of the proposed impedance matrix interpolation scheme can result in significant savings in the computation time with little or no compromise in the accuracy of the solution.**

I. INTRODUCTION

Field Solvers based on the Method of Moments (MoM) are often used for the analysis and design of a wide variety of antennas and arrays [1]. However, as the problem geometry becomes large, the computation of the MoM impedance matrix elements consumes a considerable portion of the total solution time because this computation requires O(N^2) operations, where N is the number of unknowns, and must be repeated at each frequency. Though there are a number of ways [2] one can speed up the matrix solution, especially as the frequency is swept, we concentrate on the matrix generation problem as we sweep the frequency in this paper.

One promising approach to speeding up the time for matrix generation is the impedance matrix interpolation. The concept of the impedance matrix interpolation was first proposed by Newman [3] for the computation of the input impedance of a straight dipole antenna. Later, Rahmat-Samii *et al.* have improved the interpolation algorithm further for the analysis of antennas and the prediction of the response of Frequency Selective Surfaces (FSSs) [4-5]. However, in these papers the interpolation scheme has been employed only for structures in free space, *e.g.*, freestanding FSSs or antennas, and the performance of this scheme when applied to planar microstrip structures has never been examined.

In this paper, we present a new matrix interpolation scheme using three different interpolating functions in three different regions and demonstrate its efficacy by analyzing a variety of planar microstrip structures.

II. NEW IMPEDANCE MATRIX INTERPOLATION ALGORITHM

To understand the behavior of the impedance matrix elements for microstrip structures as functions of the frequency, let us consider a mixed potential integral equation (MPIE) formulation presented in [1]. In this formulation, the impedance matrix elements of the MoM can be separated into three terms in accordance with their contributions to the matrix elements. The first of these corresponds to the contribution of the magnetic vector potential and is related to a magnetic vector Green's function. The second one is from the contribution of the electric scalar potential and is related to the electric scalar Green's function. Finally, the third one is the contribution of the ohmic losses. We note that the frequency variations of the impedance matrix elements are intimately related to those of the above Green's functions, and we take advantage of this fact to postulate the frequency behaviors of the matrix elements. Note that there is a $j\omega$ factor contained in the first term, and a $1/j\omega$ factor in the second term, and these behaviors are reflected in the matrix elements as well. The behaviors of the Green's functions for a microstrip structure have been investigated by many researchers [6]. It has been determined that the magnitude of the vector Green's function changes little with frequency, while the corresponding variation of the scalar Green's function depends on the distance. In the near region, the scalar Green's function varies little with frequency, while its magnitude is proportional to f^2 for the intermediate region, where f is the frequency; also, in the far region, it is proportional to f^4. Hence, we need to use different interpolating functions depending on the distance between the source and the testing functions. In the intermediate and far regions, the phase varies rapidly as a function of frequency and needs to be factored out before we begin the interpolation process.

When working with microstrip type structures, we need to use a normalized distance $k_e\rho$ to describe the behavior of the Green's function, where k_e is the effective wave number of the microstrip structure, and ρ is the radial distance in the x-y plane. We have carried out extensive numerical experiments with microstrip structures and have identified the following distance criteria on the basis of these experiments: $\rho_0 = 0.45\lambda_e$ for transitioning from the near to the intermediate region, where λ_e is the effective wavelength; and $\rho_1 = 0.75\lambda_e$ for moving from the intermediate into the far region.

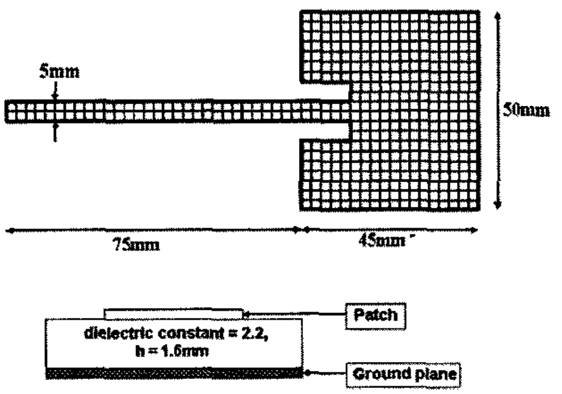

Fig. 1. Geometry of a patch antenna fed by a microstrip line.

A microstrip line-fed patch antenna, shown in Fig. 1, is considered to illustrate the behavior of the impedance matrix elements as functions of the frequency. Figure 2 shows the impedance matrix of typical elements of the patch antenna as functions of the frequency, ranging from 1 to 5 GHz, for three different distance cases. The current on the antenna is modeled by using rooftop basis functions. All three elements correspond to those elements for the x-directed rooftop basis and testing functions. For the near region, where the distance between the basis and the testing functions is relatively small ($\rho_{mn} < 0.45\lambda_e$), Fig. 2 shows that the imaginary part of a typical impedance matrix element is much larger than the real part, which decreases inversely as a function of frequency (f^{-1}), and its phase variation is very small. We interpolate the imaginary part by using an inverse f function in f^{-1}, which contains both the linear and inverse terms, given by

$$X(f) = A + \frac{B}{f} + Cf \qquad (1)$$

, and the real part by using a quadratic interpolating function ($A + Bf + Cf^2$). It is noticeable that a quadratic interpolating function is always found to be adequate for the real part over the entire region.

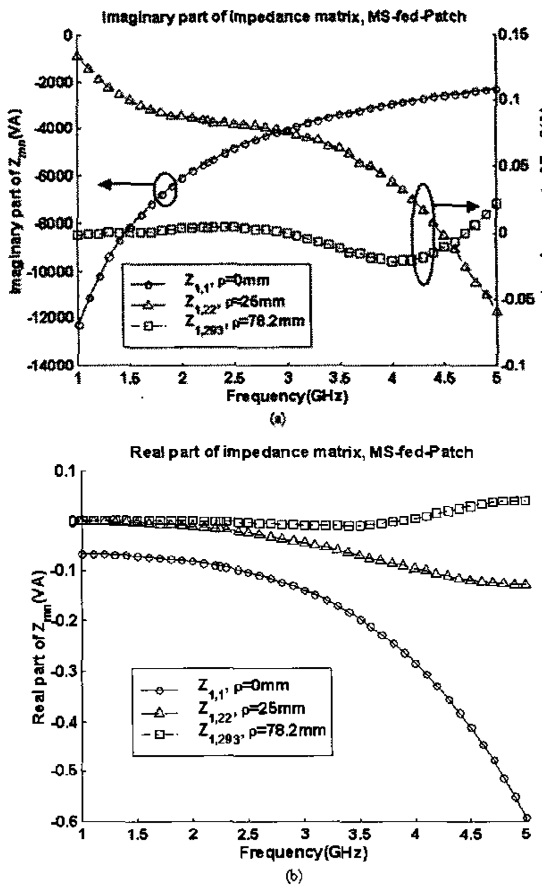

Fig. 2. Impedance matrix elements of the microstrip line-fed patch antenna for three different distances: (a) imaginary part and (b) real part.

For the intermediate region ($0.45\lambda_e \le \rho_{mn} \le 0.75\lambda_e$), we observe an increase in the magnitude of the real parts of the matrix elements, as well as their phase variations, and the phase variations dominate the frequency variations of the impedance matrix elements. To circumvent this problem, we factor out this term and interpolate only the rest, which is relatively slowly varying, and recover the original impedance matrix elements later by restoring the phase factor to the interpolated result. In this region, we use the quadratic function below for both the real and imaginary parts of $Z_{mn} / e^{-jk_e\rho_{mn}}$:

$$X(f) = A + Bf + Cf^2 . \qquad (2)$$

For the far region ($\rho_{mn} > 0.75\lambda_e$), the real parts of the impedance matrix now become comparable to the imaginary parts and the influence of the phase term $e^{-jk_e\rho_{mn}}$ becomes more severe and the elements fluctuate more rapidly as functions of the frequency. Similar to the intermediate region, the phase term is again factored out and only the remaining is interpolated. For this region, we use the cubic function below for the imaginary parts of $Z_{mn}/e^{-jk_e\rho_{mn}}$, *i.e.*, we let:

$$X(f) = Af + Bf^2 + Cf^3 \quad (3)$$

while the quadratic function is still used for the real parts of $Z_{mn}/e^{-jk_e\rho_{mn}}$.

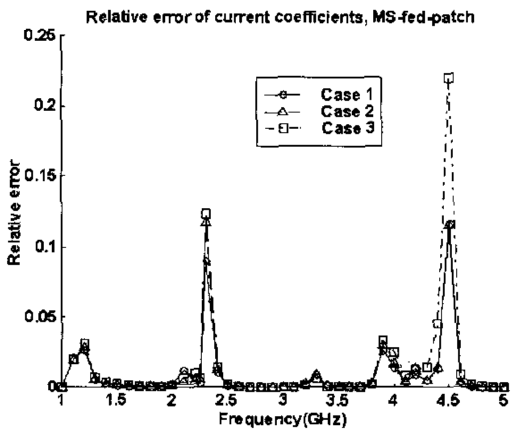

Fig. 3. Comparison of relative errors of current coefficients using different interpolation schemes for the microstirp line-fed patch antenna.

To demonstrate the efficacy of the proposed method, we have compared the current coefficients obtained from our interpolation scheme with two others [4-5] against the results of the direct computation. Three different interpolation schemes including the proposed one are implemented from 1 to 5 GHz for the patch antenna. For the first case, the proposed interpolation scheme is employed. For the second case, we employ the inverse f and quadratic functions for interpolating the imaginary and real parts, respectively, over the entire region. For this case, the distance criterion for factoring out the phase term is $0.5\lambda_e$. For the last case, we experiment with the same interpolating function as above except for the imposition of the distance criterion of $0.5\lambda_o$. Figure 3 shows the error norm for the current coefficients for three cases. The results clearly show that the scheme we have proposed works better than the other two existing schemes.

III. NUMERICAL RESULTS

The three-region interpolation scheme, described above, has been applied to a patch antenna, fed by a microstrip line as shown in Fig. 1. A modeling of this antenna at the highest frequency of 5 GHz with a cell size of 2.5mm requires 730 unknowns when a uniform rectangular grid and rooftop basis functions are employed, and it remains unchanged over the frequency range of interest. The proposed interpolation scheme is implemented from 1 to 5 GHz, and two different frequency step sizes are used for this case.

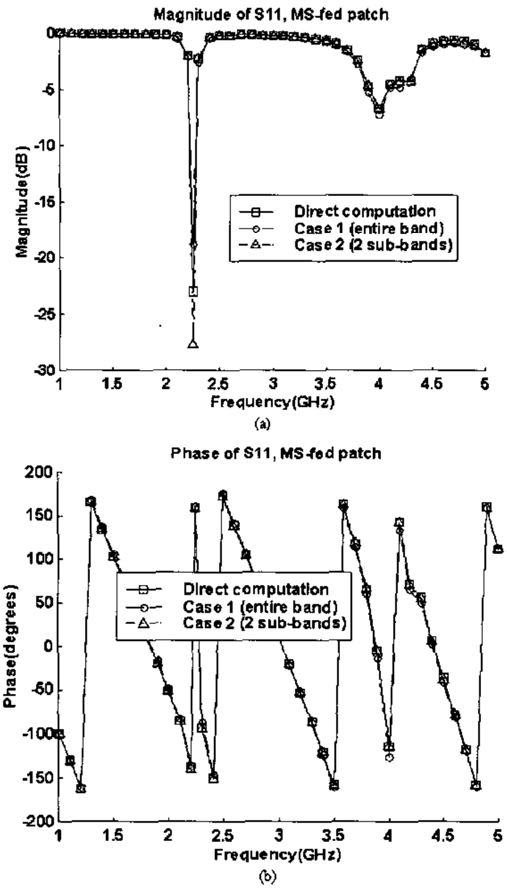

Fig. 4. Comparison of the S11 characteristic for the microstrip line-fed patch: (a) magnitude; and (b) phase.

For the first case, the frequency step size $\Delta f = 2$ GHz is employed, and the matrices are directly computed and stored at 1, 3, and 5 GHz. For the second case, we use a frequency step of 1 GHz, so that the entire frequency band is divided into 2 sub-bands, *viz.*, 1 to 3 GHz and 3 to 5 GHz. The impedance matrices at 1, 2, and 3 GHz are pre-calculated directly for the first sub-band, while at 3, 4, and

5 GHz for the second sub-band. The S11 characteristics for the two cases are compared at every 0.1 GHz including the first resonant frequency 2.25 GHz, as shown in Fig. 4, while Fig. 5 compares the radiation patterns at 2.25 GHz. We observe that the error of the S_{11} for the second case, which uses a frequency step size of 1 GHz, is much lower than that for the case where the frequency step size was 2 GHz.

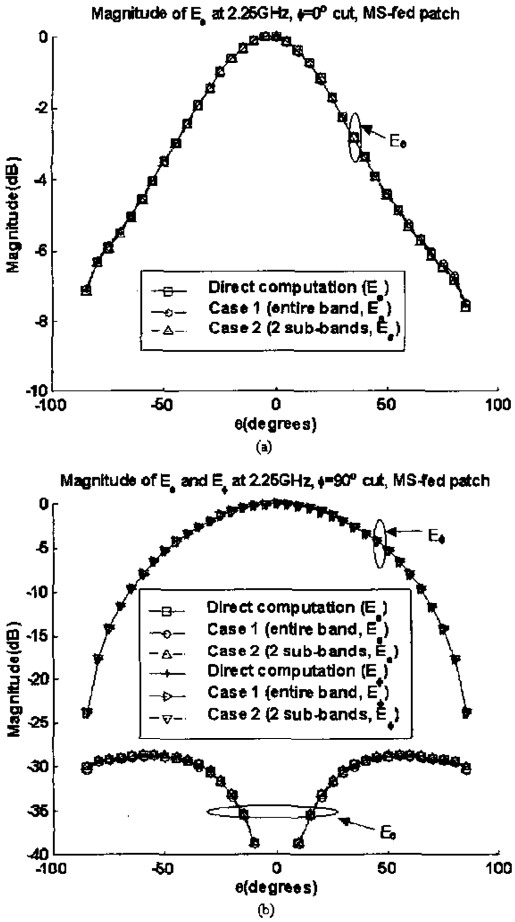

Fig. 5. Comparison of the radiation patterns at 2.25 GHz for the microstrip line-fed patch: (a) ϕ=0° cut; and (b) ϕ=90° cut.

At this point, we need to discuss how to choose an optimum interpolation frequency step size [2]. Since we use the effective wavenumber and wavelength, the corresponding wavenumber step size is $\Delta k_e = 2\pi/\lambda_e$. We require the interpolation step size to introduce a phase change of less than π; hence $\Delta k_e \rho_{mn} \leq \pi$. Let us assume that the largest distance between the source and testing functions as $\rho_{\max}$. From this, we can derive the maximum interpolation step size to be $\Delta f_M = \dfrac{f_H}{2(\rho_{\max}/\lambda_e)}$ where f_H is the upper-limit of the frequency band. It turns out that a frequency step size of less than Δf_M is a good guideline for achieving a close agreement with the results of direct computation. For this example, the largest distance $\rho_{\max}$ is about $2.5\lambda_e$ at the highest frequency and Δf_M is about 1 GHz. As we observe from Figs. 4 to 5, the interpolation result with a frequency step size of 1GHz shows good agreement when compared to the direct computation. The time for the direct impedance matrix calculation is 6.6 seconds per frequency on a Pentium III PC with 550MHz processor and 1GByte RAM, whereas it is 0.42 seconds using the interpolation scheme on the same machine.

IV. Conclusions

In this paper, we have presented a new interpolation technique for MoM matrices associated with planar microstrip structures. We have identified three different regions, *viz.*, near, intermediate, and far, on the basis of the distance between the source and the testing functions, and have presented different interpolating schemes tailored for each region. The matrix interpolation scheme is extendable to a general class of problems that are currently under investigation by the authors.

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